New topics?
J.C. MAXwell

# History of Mathematics

11:00 – 1:00 T R in UW2–305

Office hours 9:45 – 10:45 UWBB 223

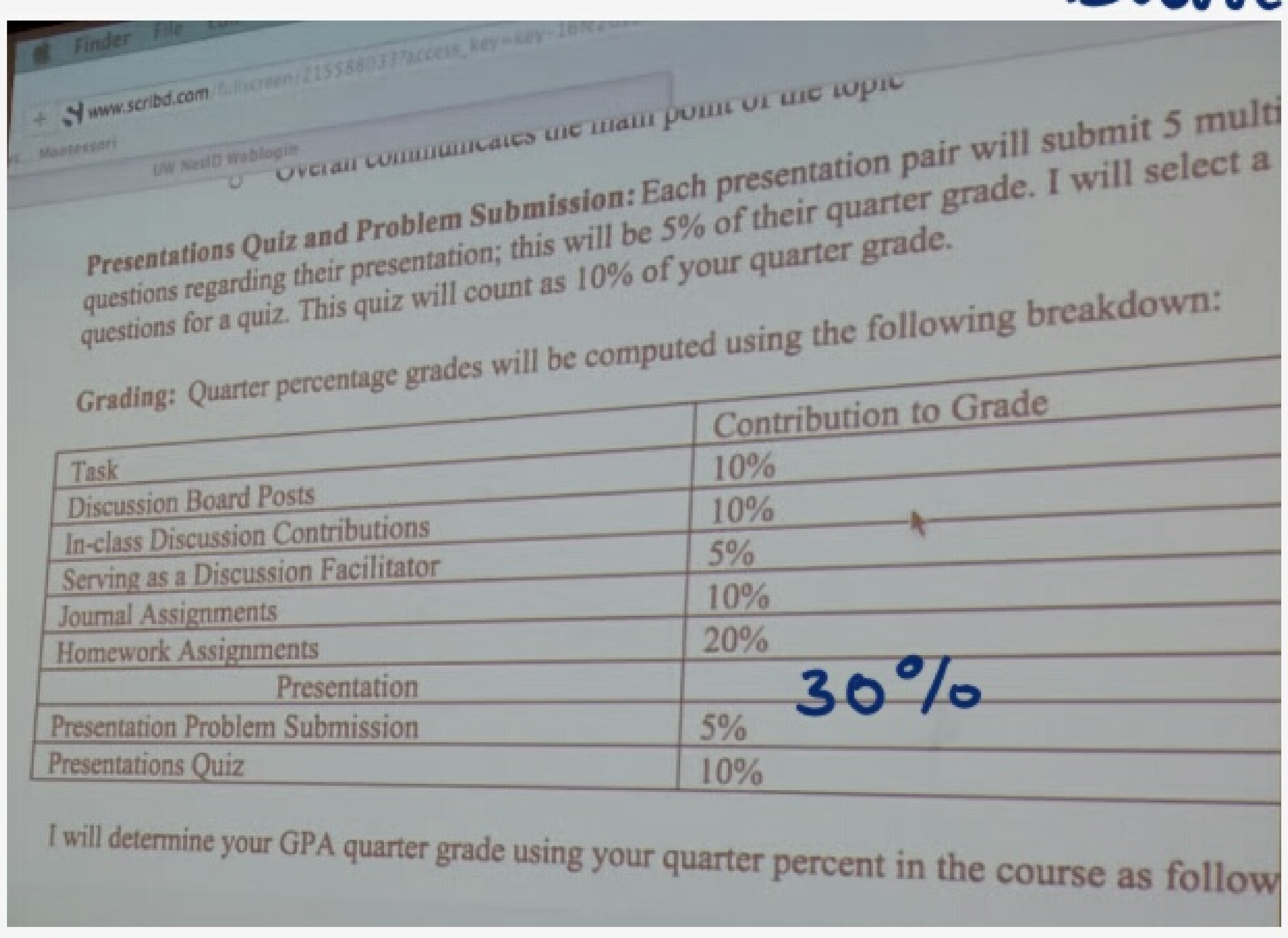
## The History of Mathematics, An Introduction

1. Discussions about great mathematicians
2. Utilize tools of a time to solve problems
3. Prove the proofs
4. Describe the mathematics of a particular topic

- Facilitator moves the discussion along.
- Hella read your very expensive book
- 30/40 minute readings // use mathematics of the time.

Math Club
Thursday, April 3
1530–1830
UWBB 216

- Grade for: discussions in class
  10% discussion Board posts
  Facilitator role
  Homework exercises from book
  30% Presentations in pairs.
  ↳ 45-50 minute prezi → History - Setting
  → Mathematician - Who?
  → Math involved
  ↳ start from book



www.scribd.com

Overall communicates the main point of the topic

**Presentations Quiz and Problem Submission:** Each presentation pair will submit 5 multi[ple] questions regarding their presentation; this will be 5% of their quarter grade. I will select a[...] questions for a quiz. This quiz will count as 10% of your quarter grade.

**Grading:** Quarter percentage grades will be computed using the following breakdown:

| | Contribution to Grade |
|---|---|
| Task | 10% |
| Discussion Board Posts | 10% |
| In-class Discussion Contributions | 5% |
| Serving as a Discussion Facilitator | 10% |
| Journal Assignments | 20% |
| Homework Assignments | |
| Presentation | 30% |
| Presentation Problem Submission | 5% |
| Presentations Quiz | 10% |

I will determine your GPA quarter grade using your quarter percent in the course as follow[s]

☐ By 1500, must post discussion via email

Babylonian Mathematics

3500 B.C. - 530 B.C.

Cuneiform - clay tablets (preserved well in desert)

3,000 BC to 75 AD

= 1

= 10

Base 60

sexegesimal

Positional number system

Developed in time of Hammurabi

45 =

Base 60:

$\_\_ 60^3 + \_\_ 60^2 + \_\_ 60^1 + \_\_ 60^0$

Thus $84 = 1\ 60^1 + 24\ \_\ 60^0$ → - 84 but also

$1 \cdot 60^1 + 24 \cdot 60^0$

3624

$(1 \cdot 60^2 + 0 \cdot 60^1 + 24 \cdot 60^0)$

divider

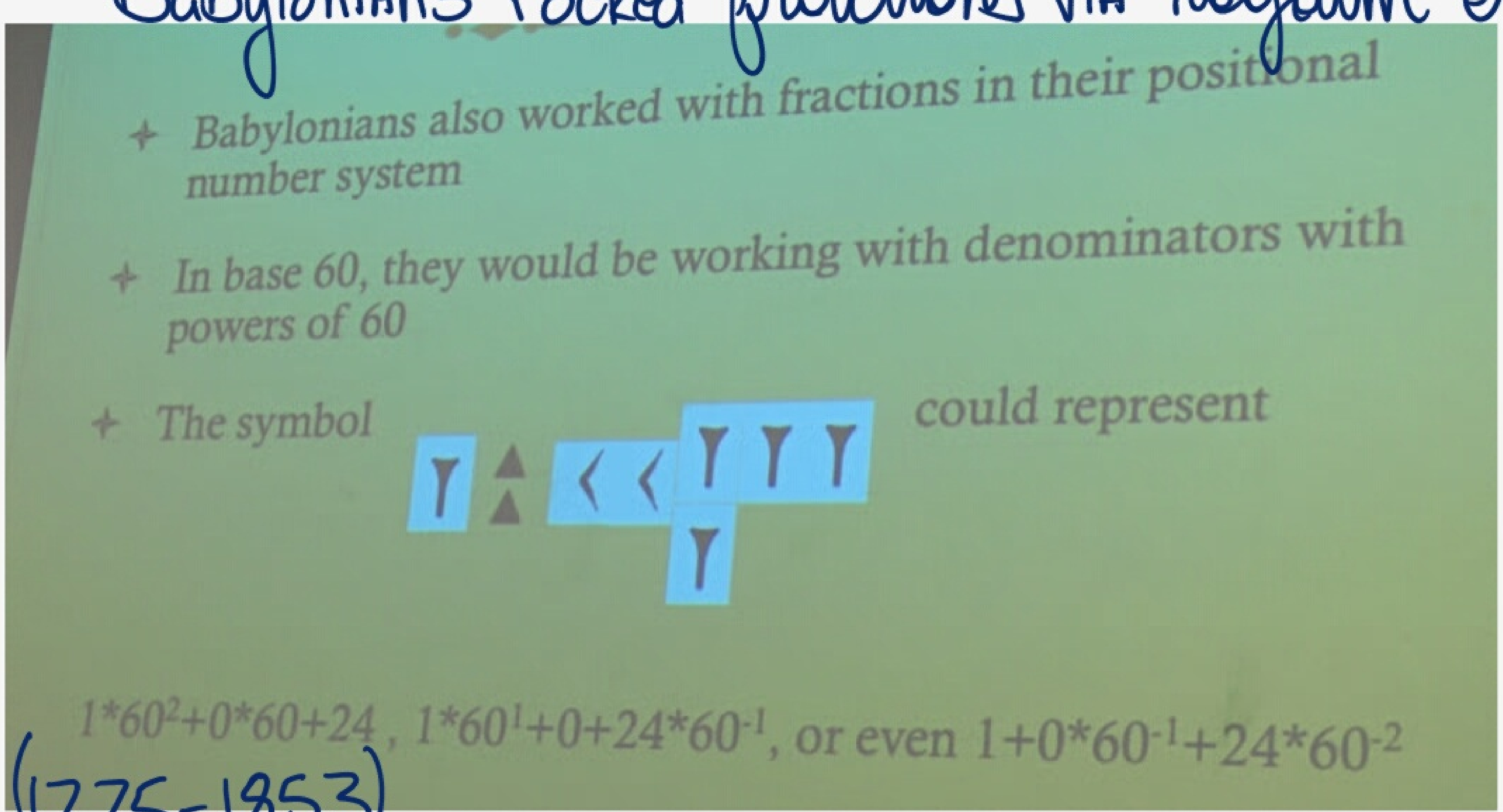
≈ 300 B.C.

↳ represents Ø of "this" power

Claudius Ptolemy (90 AD - 158 AD) (O)

↳ Began to use the omicron to represent an empty placeholder

Babylonians rocked fractions via negative exponents of 60.




star Calendar

360 days

but 360 too high.

60 divides 360

60 has many divisors.

Helped w/ Fractions

(1775-1853)

Grotefend → on a drunken wager, bet he could decipher cuneiform.

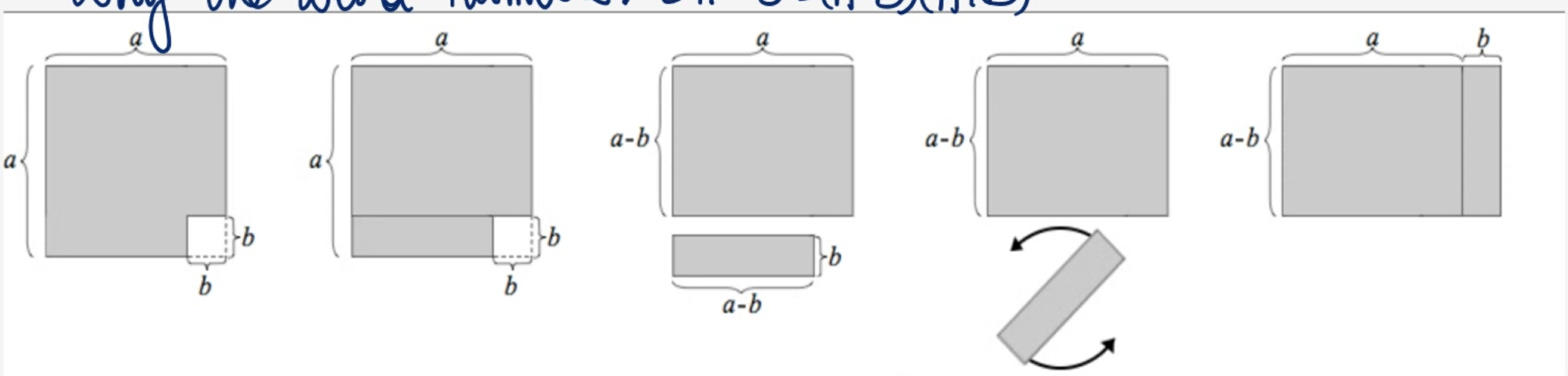
↳ Dismissed as an ameteur.

"Mountain of the Gods" translated by Rawlinson

↳ Commisioned by Darius the Great

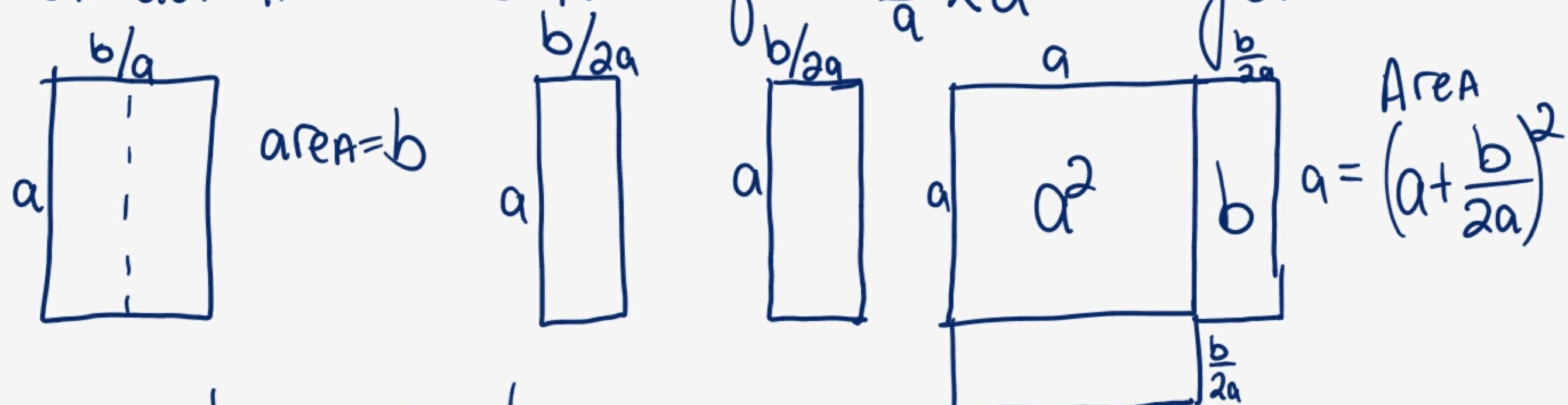
- Plimpton 322 dated 1900 BC-1600 BC
  - ↳ $\text{Leg}^2 + \text{Hypoteneuse}^2$ is an integer! integral solutions to pythagorean thm.
  - ↳ Over 1000 years before Pythagorus
  - ↳ exhibits some scribal error } Needed → Geometric Proof

Why the weird numbers. $A^2-B^2=(A-B)(A+B)$



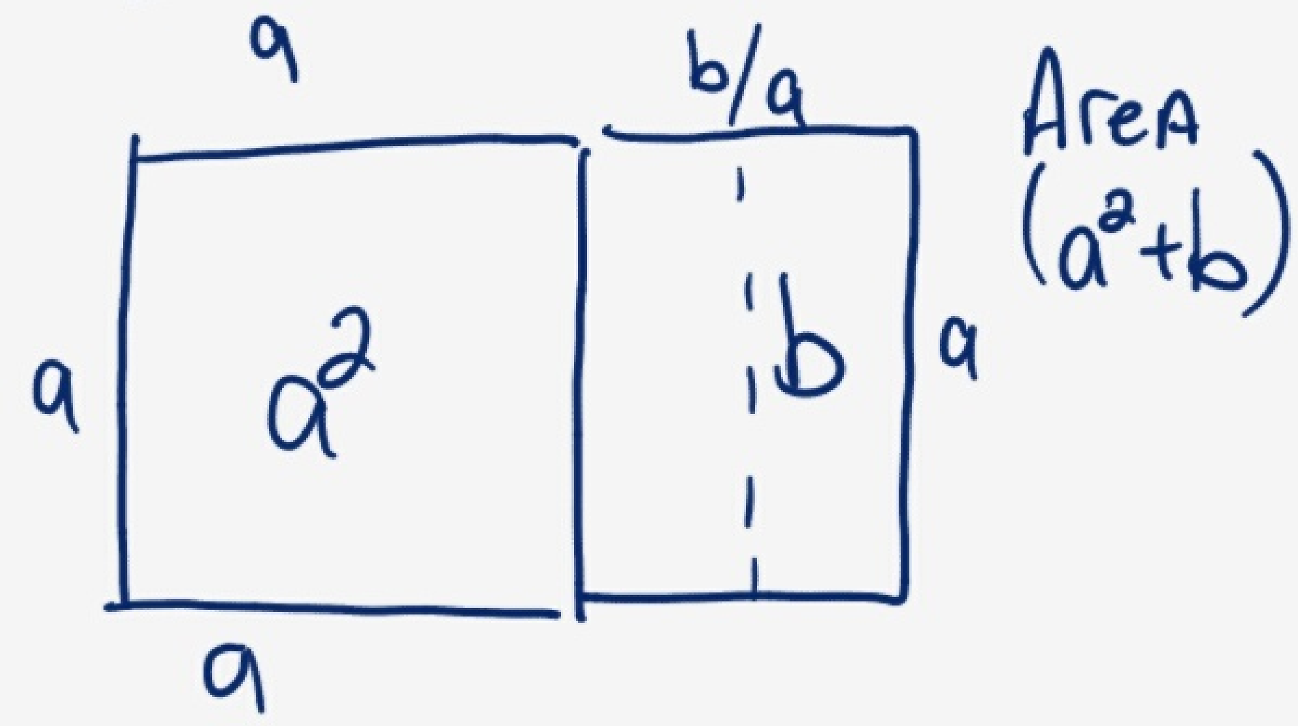

Babylonians knew sqrt approx $\sqrt{a^2+b} \approx \left(a+\frac{b}{2a}\right)$

- To approximate $\sqrt{N}$, choose a s.t. $a^2<N$, but it is close
- Consider the area $b=N-a^2$ of an $\frac{b}{a} \times a$ rectangle.



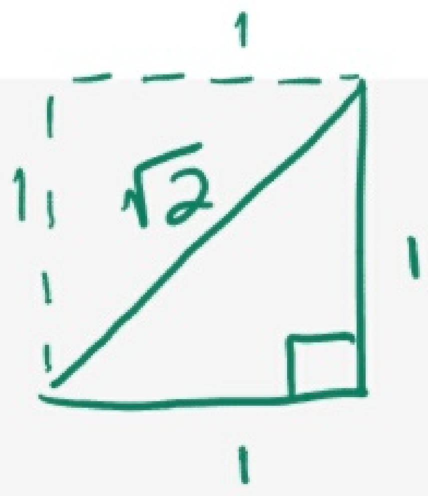

## Section 3.4

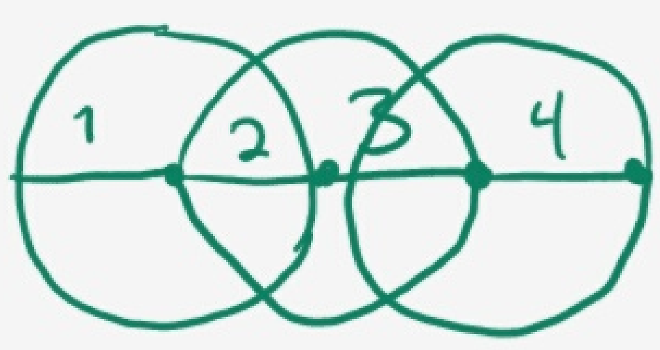
↳ discussion

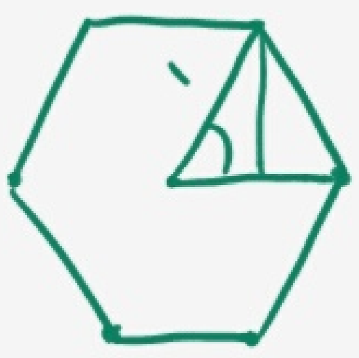


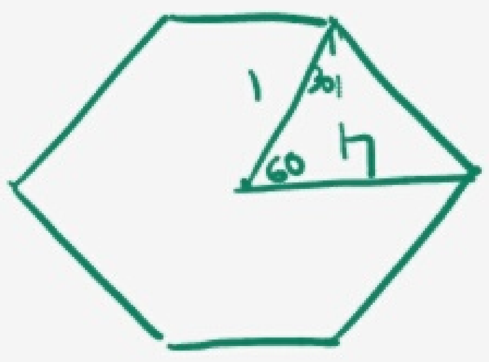
# Quadrature of the Circle
# "Squaring the Circle"

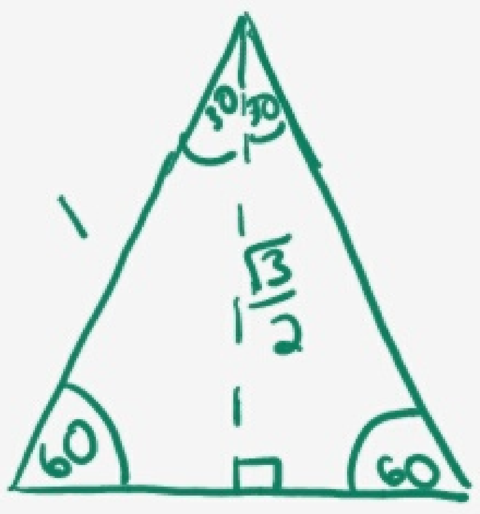
- Construct a square whose area is equal to the area of a given circle
- Area circle radius $r = \pi r^2$
- Area square = $l^2$
- Want to construct $l$ such that $l = \sqrt{\pi r^2} = r\sqrt{\pi}$








# More constructible numbers

- Positive integers
- Adding, subtracting, multiplying, and dividing constructible numbers yields another constructible number.
- The square root of a constructible number is a constructible number.

$\pi$ cannot be constructed

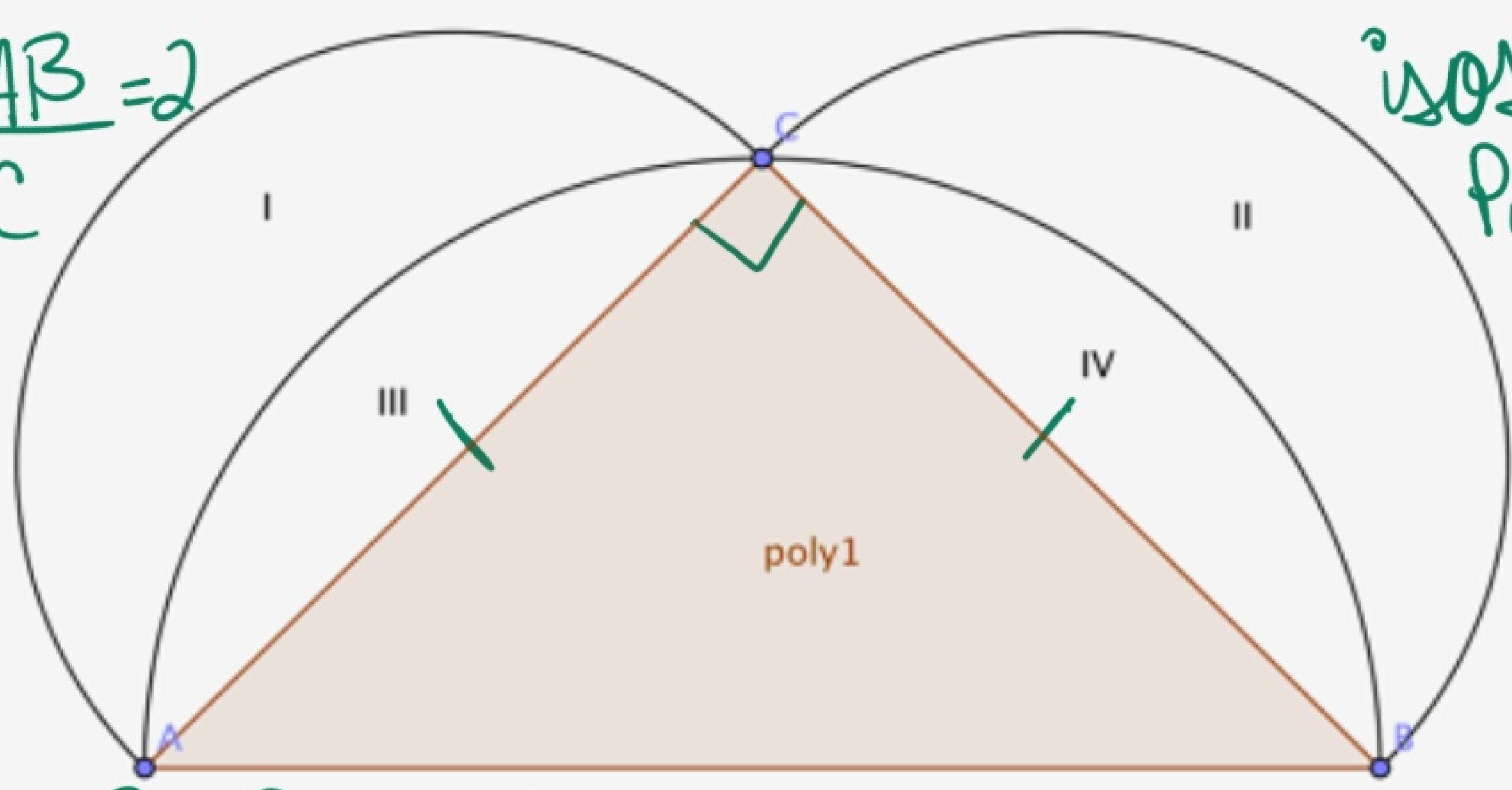
$\sqrt[3]{2}$ cannot be constructed

# Hippocrates of Chios (460-380 B.C.)

- Studied the *impossible* construction problems
  - Squaring the circle
  - Doubling the cube
  - Trisecting an angle
- He made significant contributions to area
  - Squaring a lune
  - Reducing doubling cube problem

Geometry fact:

$$\frac{\text{Area of semi } AB}{\text{Area of semi } AC} = 2 = \frac{AB^2}{AC^2}$$




since ABC right isosceles triangle. Pythagoras tells us $AB^2 = AC^2 + CB^2$

But $AC = CB$ so $AB^2 = 2AC^2$

↳ $2 = \frac{AB^2}{AC^2}$

thus

area1 + area2 = Area $\Delta ABC$

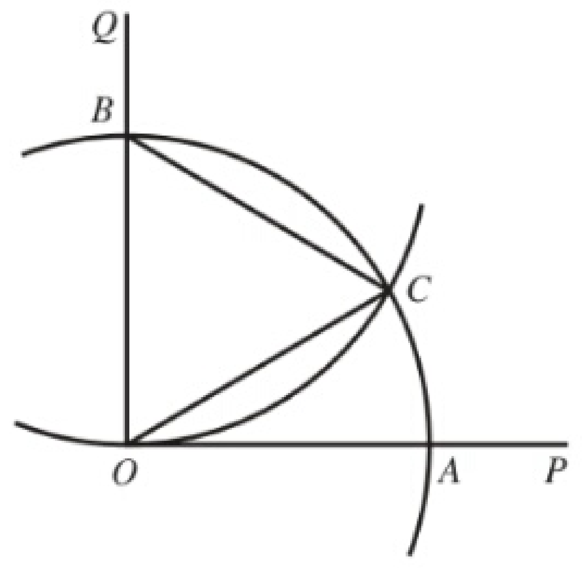
But area1 = area2 → Area $\Delta ABC$ = 2 area1 ↗ $BC = AC$!

↳ area1 $= \frac{1}{2}$ Area $\Delta ABC = \frac{1}{2}\left(\frac{1}{2} BC \cdot AC\right) = \left(\frac{AC}{2}\right)^2 = \frac{1}{2}(\text{area } \Delta ABC)$

($\frac{1}{2}$ Base·Height)

## The Trisection of an Angle

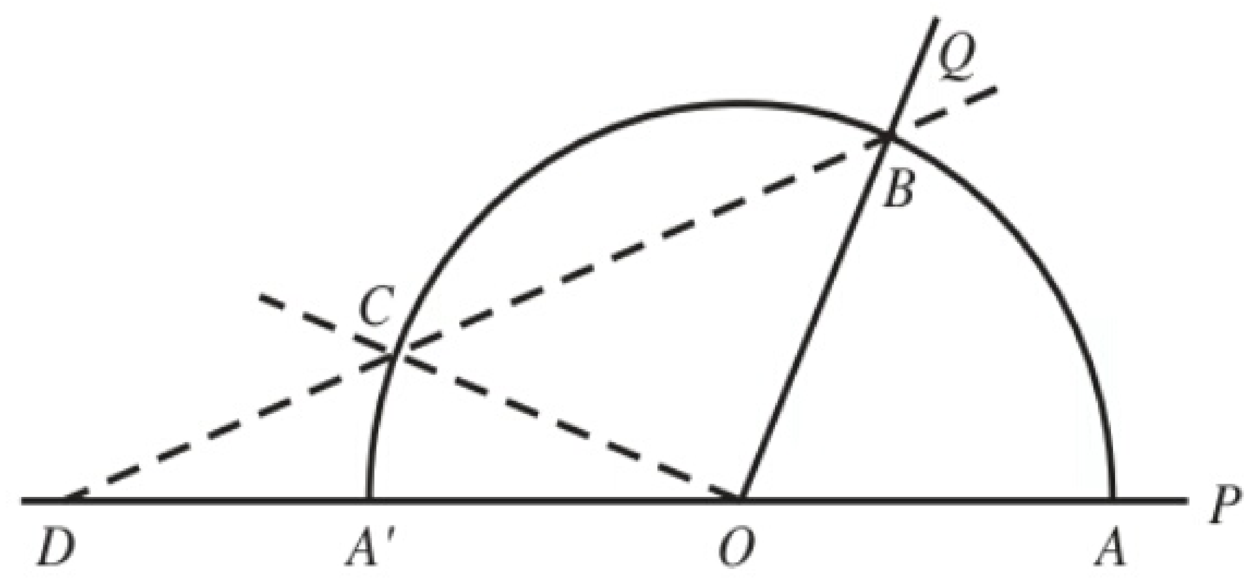
Although Hippocrates advanced two of the three famous construction problems, he made no progress with trisecting an angle. The bisection of an angle with only straightedge and compass is one of the easiest of geometrical constructions, and early investigators had no reason to suspect that dividing an angle into three equal parts under similar restrictions might prove impossible. Some angles can obviously be trisected. In the special case of the right angle $POQ$, the construction is found as follows. With $O$ as a center, draw a circle of any radius intersecting the sides of the angle in points $A$ and $B$. Now draw a circle with center at $B$ and passing through $O$. The two circles will intersect in two points, one of which will be a point $C$ in the interior of angle $POQ$.



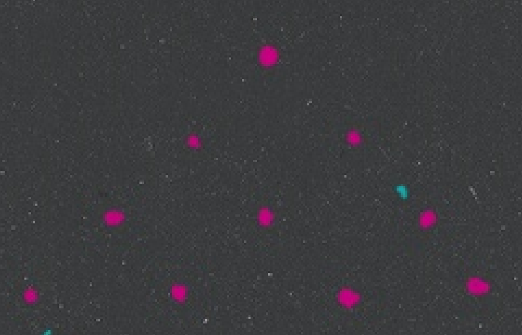

Triangle $BOC$ is equilateral, hence equiangular; therefore $\angle COB = 60^\circ$. But then

$$\angle COA = 90^\circ - 60^\circ = 30^\circ = \tfrac{1}{3}(90^\circ),$$

and line $OC$ is a trisector of the right angle.



~600 B.C. Greece → Pythagoras



fire
water
air
earth

tetratys

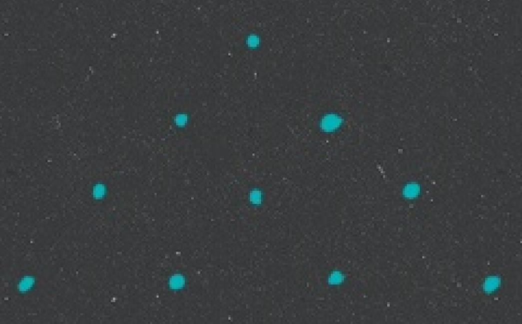
Pythagoreans were friendly vegans

- 569 B.C. ish, Samos
- At 18 → left to study in Phoenicia & Egypt
- Babylonia until 50
- returned to found a school in Crotana (Italy)
  - ↳ 4 mathemata

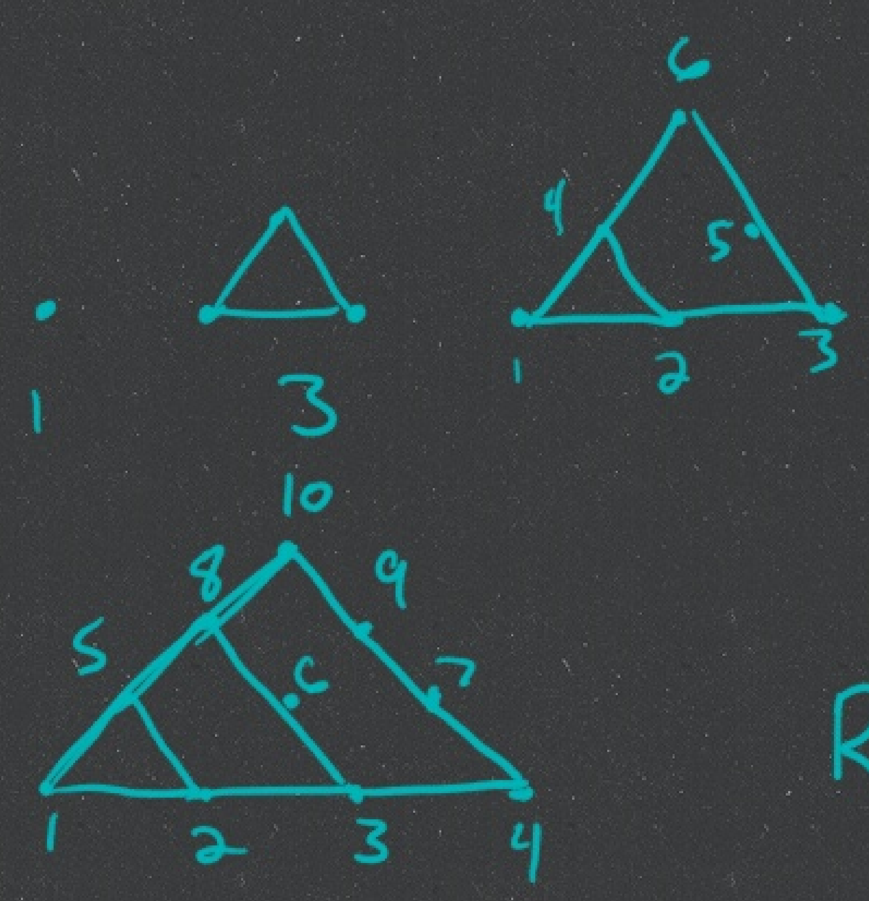
$\underbrace{\text{· arithmetica · harmonia · geometria · astrologia}}_{\text{Quadrivium}}$ > · logic · grammar · rhetoric

→ liberal arts of Ancients

- Theano, woman of math w/ close relationship



triangular numbers



$t_1=1$
$t_2=3$
$t_3=6$
$t_4=10$

Triangular # Def/ # of vertices in an equilateral triangle with side length n-1 is called the $n^{th}$ triangular number, denoted $T_n$.

Recursive Formula: $t_n = t_{n-1} + n$

→ L shape summation

$$\sum_{i=1}^{n} (2i-1) = n^2$$

$$t_n = t_{n-1} + n$$

$$= (t_{n-2}) + (n-1) + n$$

$$= (t_{n-3}) + (n-2) + (n-1) + n$$

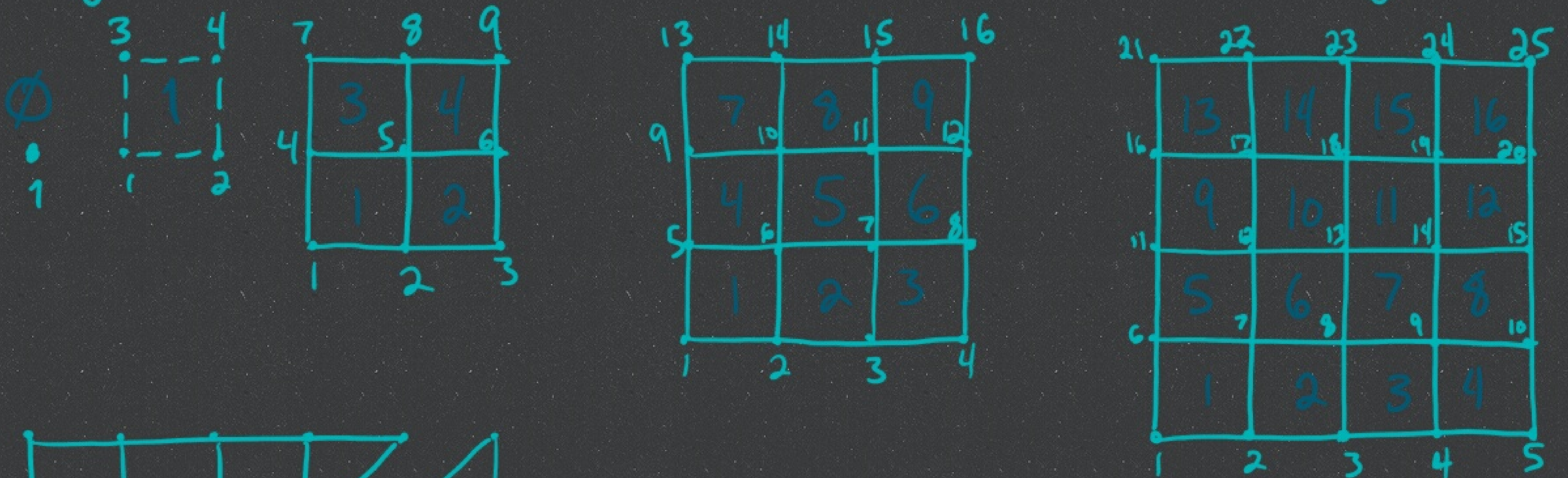
$$\vdots$$

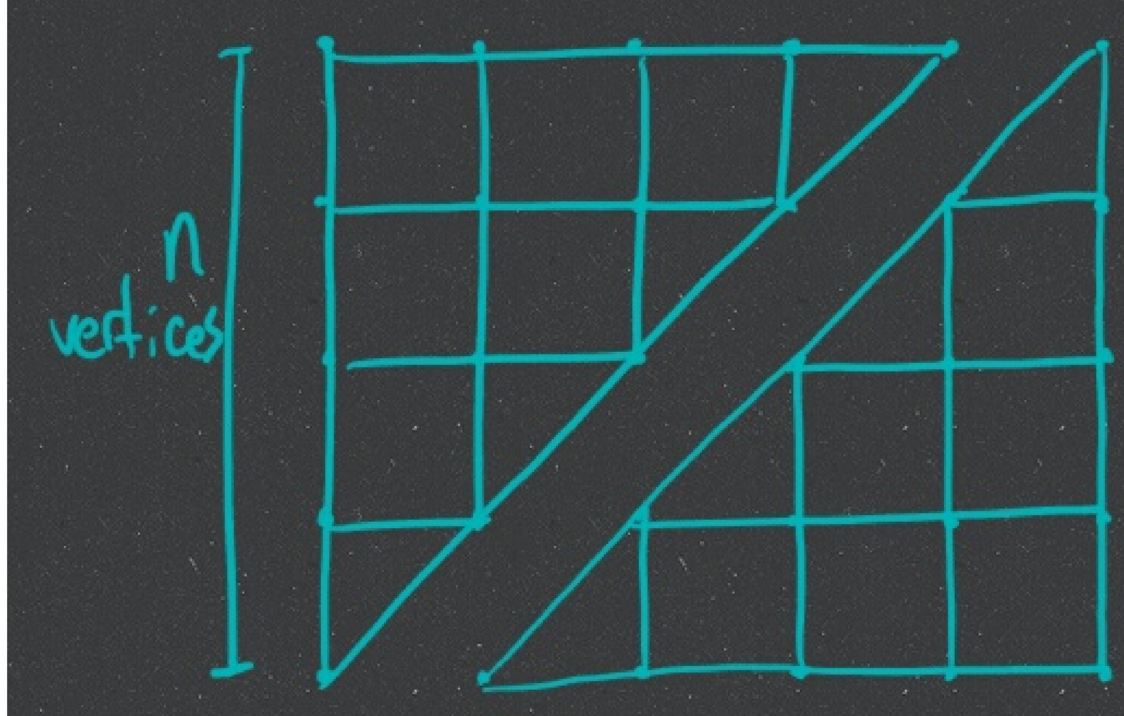
$$= t_1 + 2 + 3 + \cdots + (n-3) + (n-2) + (n-1) + n$$

$$= 1 + 2 + 3 + \cdots + (n-3) + (n-2) + (n-1) + n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

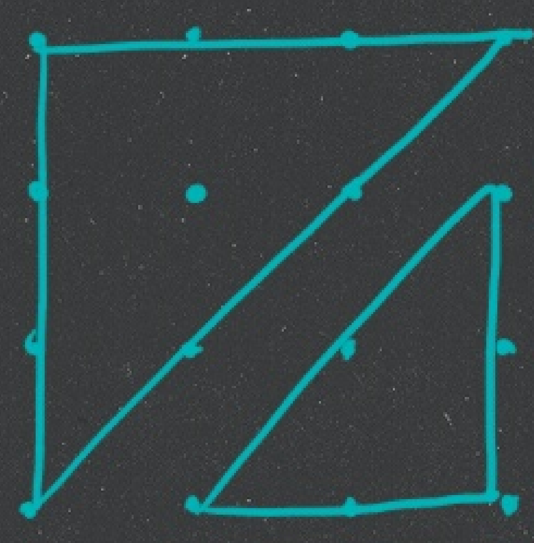
Square numbers - a number which is the number of vertices in a square.






•Chapter 3.2
in H.O.M. book•

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

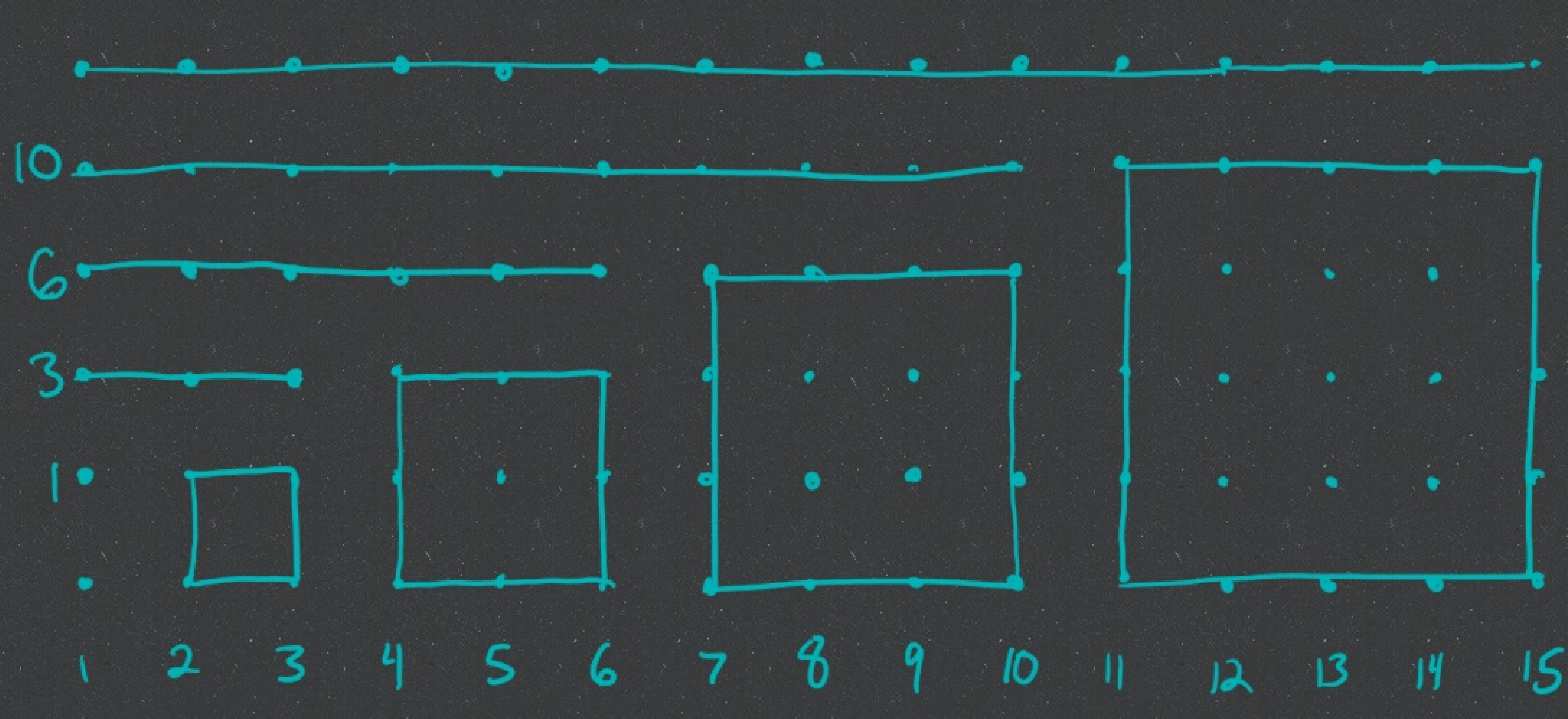


$$S_n = t_n + t_{n-1}$$

$$S_n = \frac{n(n+1)}{2} + \frac{(n-1)(n)}{2}$$

$$\frac{1}{2}\left(n^2+n+n^2-n\right)$$

$$\frac{1}{2}\left(2n^2\right) = n^2$$




# of vertices in square
+ # of vertices in line segments
= # of vertices in rectangle

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

sum of
- first n numbes
- first n odd
- first n squares
- first n cubes

Skip Lester

Translate into our number system:

2(b)

$$32\cdot 60^2 + 25\cdot 60^1 + 12\cdot 60^0$$

$$\begin{array}{r} 3600 \\ \times\ 32 \\ \hline 7200 \\ 10800\text{x} \\ \hline 115200 \end{array} \qquad \begin{array}{r} 25 \\ \times 60 \\ \hline 0 \\ 150\text{x} \\ \hline 1500 \end{array}$$

$$115{,}200 + 1500 + 12$$

$$\begin{array}{r} 1500 \\ +\ 12 \\ \hline 116712 \end{array}$$

$$= \underline{116{,}712}$$

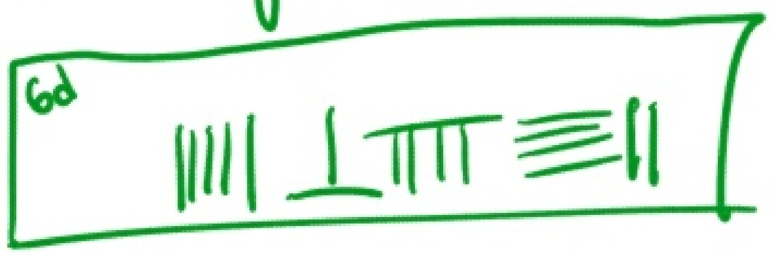
4) (d)

$$1,23;45 = 1\cdot 60^1 + 23\cdot 60^0 + 45\cdot 60^{-1}$$

$$= 60 + 23 + \frac{45}{60} = 73\tfrac{3}{4} = \underline{\underline{73.75}}$$

(6)(d) Bamboo numbers for 57,942

The author's table doesn't make sense so I couldn't understand his example. This is my best guess from book's info.




**5.** Multiply the number 12,3;45,6 by 60. Describe a simple rule for multiplying any sexagesimal number by 60; by $60^2$.

$$60\left[(12\cdot 60^1) + (3\cdot 60^0) + (45\cdot 60^{-1}) + (6\cdot 60^{-2})\right]$$

$$= (12\cdot 60^2) + (3\cdot 60^1) + (45\cdot 60^0) + (6\cdot 60^{-1})$$

$$= \underline{43425.1}$$ ; shift the semicolon one place left to multiply by $60^1$. Two places to multiply by $60^2$; n places for $60^n$.

$$\begin{array}{r} 3600 \\ 12 \\ \hline 7200 \\ 3600\text{x} \\ \hline 43200 \\ +\ 180 \\ \hline 43380 \\ +\ 45 \\ \hline 43425\frac{6}{60} \end{array}$$

7(d)

9 6 2 3 0 8 8

$= \underline{9{,}623{,}088}$

**8.** Multiply  (2 6 7 7 2) by 10 and express the result in Chinese rod numerals. Describe a simple rule for multiplying any Chinese rod numerals by 10; by $10^2$.

$10(26772) = 267720 =$  Add a zero ("0") to the right side of #.

Just as in our decimal system, add 2 zeros to right end of number to multiply by 100; n zeros for $10^n$.

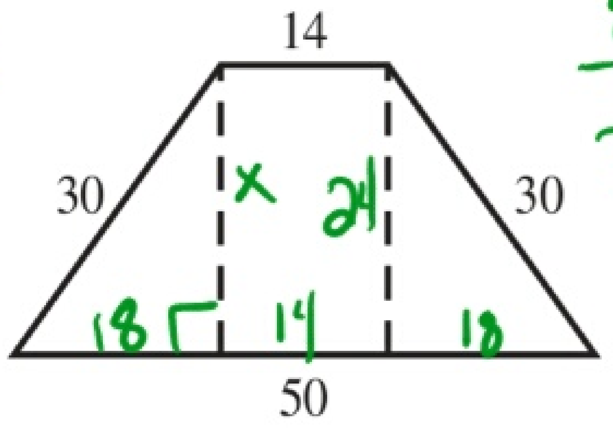
**2.** An Old Babylonian tablet calls for finding the area of an isosceles trapezoid whose sides are 30 units long and whose bases are 14 and 50. Solve this problem.

$$30^2 = 18^2 + x^2$$

$$\sqrt{30^2 - 18^2} = x$$

$$= 24$$

or use Side-Side-Angle




$$\begin{array}{r} 50 \\ -14 \\ \hline 36 \end{array} \qquad \frac{36}{2} = 18$$

$$\text{area} = 2\left[\tfrac{1}{2}(18\cdot 24)\right] + 14\cdot 24$$

$$432 + 336$$

$$= 768 \text{ units}^2$$

Skip Lester

**7.** Because $a$ is smaller than $\sqrt{a^2+b}$ when $b > 0$, whereas $a + b/a$ is larger, the Babylonian mathematician often approximated $\sqrt{a^2+b}$ by taking the average of these two values; that is,

$$\sqrt{a^2+b} \approx \frac{1}{2}\left[a + \left(a + \frac{b}{a}\right)\right]$$
$$= a + \frac{b}{2a}, \qquad 0 < b < a^2.$$

Use this formula to get rational approximations to $\sqrt{2}$, $\sqrt{5}$, and $\sqrt{17}$. [*Hint:* In the first case put $a = \frac{4}{3}$, $b = \frac{2}{9}$; in the second case, put $a = 2$, $b = 1$.]

$\sqrt{2} = \sqrt{\left(\frac{4}{3}\right)^2 + \frac{2}{9}} \approx \frac{4}{3} + \frac{2/9}{2(4/3)}$ $\quad \frac{4}{3} + \frac{2}{18\left(\frac{4}{3}\right)} = \frac{4}{3} + \frac{2}{24}$

$\frac{16}{12} + \frac{1}{12} = \underline{\frac{17}{12} \approx \sqrt{2}}$

$\sqrt{5} = \sqrt{2^2+1} \approx 2 + \frac{1}{2(2)} = \underline{\frac{9}{4} \approx \sqrt{5}}$

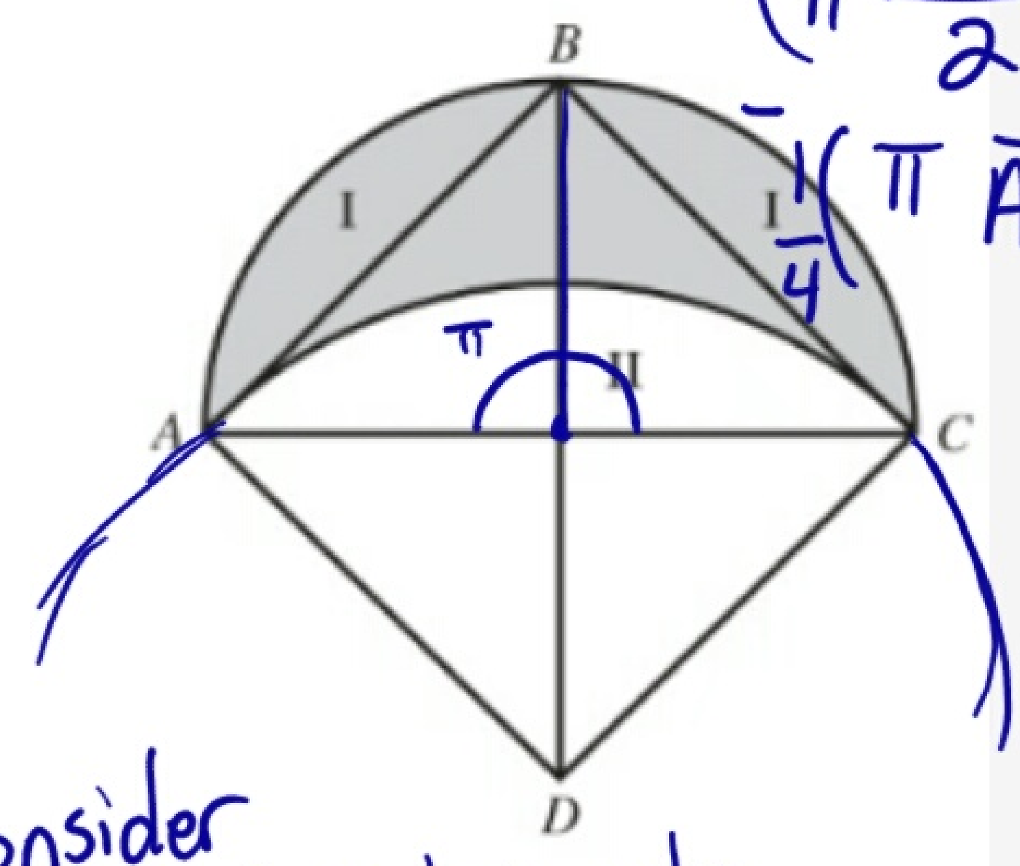
$\sqrt{17} = \sqrt{4^2+1} \approx 4 + \frac{1}{8} = \frac{33}{8} \approx \sqrt{17}$

Area $\triangle ABC$ = Area Lune ABC

## 3.4 Problems

**1.** For a variation of Hippocrates' argument that the area of a lune could be reduced to the area of a circle, begin with a square $ABCD$ and construct a semicircle on its diagonal. With the point $D$ as a center and $AD$ as radius, draw a circular arc from $A$ to $C$, as in the figure. Prove that the area of the lune, shaded in the figure, is equal to the area of triangle $ABC$. [*Hint:* Similar circular sections (the region between a chord and the arc subtended by the chord) have areas proportional to the squares of the lengths of their chords. Apply this fact to the similar sections I and II.]

$$\left(\pi \frac{\overline{AC}}{2}^2\right) - \frac{1}{4}\left(\pi \overline{AD}\right)^2$$




Consider semicircle − triangle

$$\frac{a_I}{a_{II}} = \frac{(\overline{AB})^2}{(AC)^2} = \frac{1}{2}$$

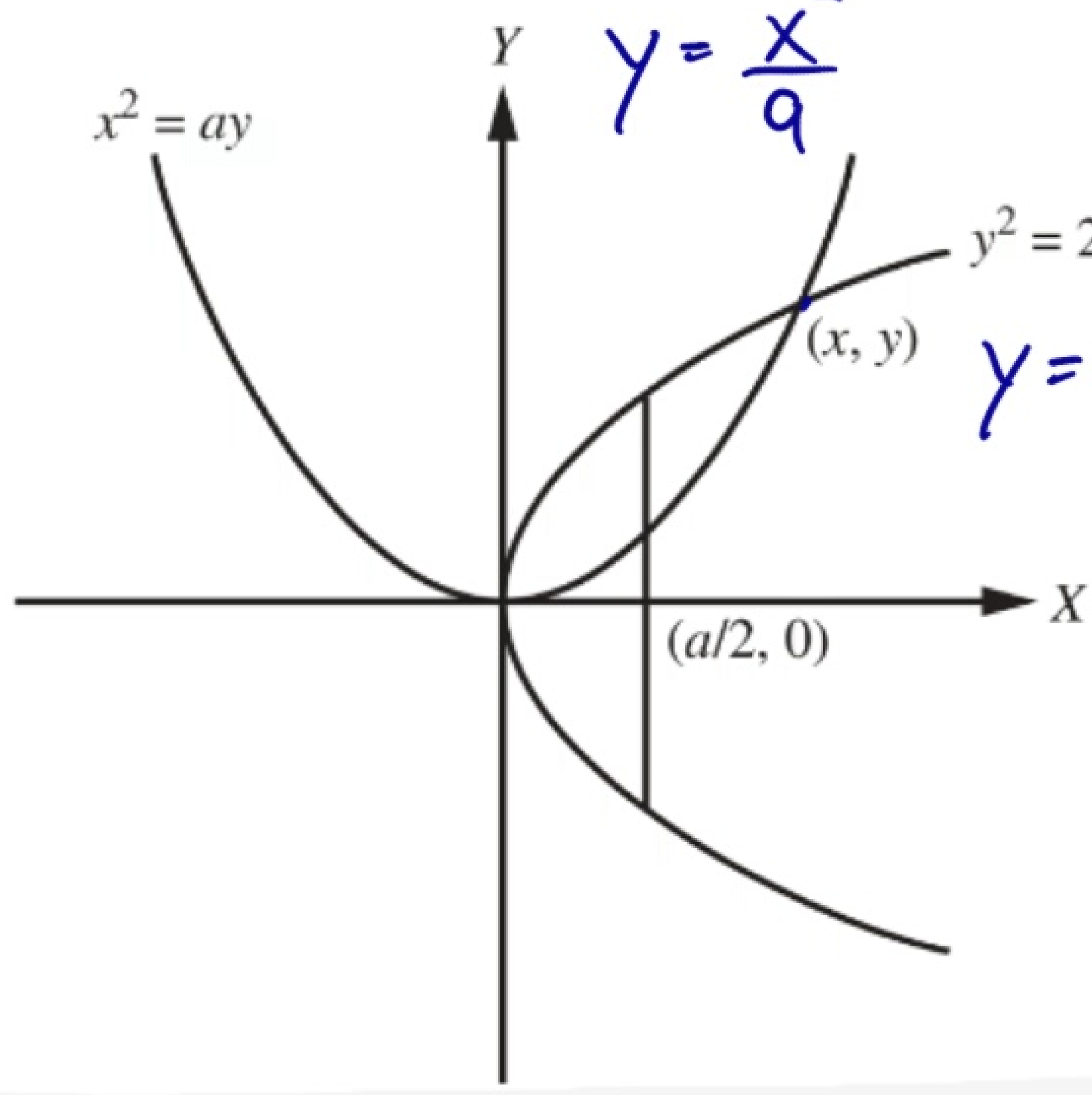
Area $ABC = \frac{1}{2} B \cdot H$

$= \frac{1}{2}(\overline{AC})(\frac{1}{2}\overline{BD})$

Note $BD = AC \rightarrow = \frac{1}{2}(\overline{AC})(\frac{1}{2}\overline{AC})$

$= \frac{1}{4}(\overline{AC})^2$ or $\left(\frac{\overline{AC}}{2}\right)^2$

**4.** The Greek mathematician Menaechmus (c. 350 B.C.), the tutor of Alexander the Great, obtained a purely theoretical solution to the duplication problem based on finding the point of intersection of certain "conic sections." To duplicate a cube of edge $a$, he constructed two parabolas having a common vertex and perpendicular axes, so that one parabola had a focal chord (latus rectum) of length $a$ and the other a chord of length $2a$. Prove that the abscissa $x$ of the point of intersection of the two parabolas satisfies the condition $x^3 = 2a^3$; the sought-for $x$, the cube's edge, is thereby obtained.




$y = \frac{x^2}{a}$

$y = \sqrt{2ax}$

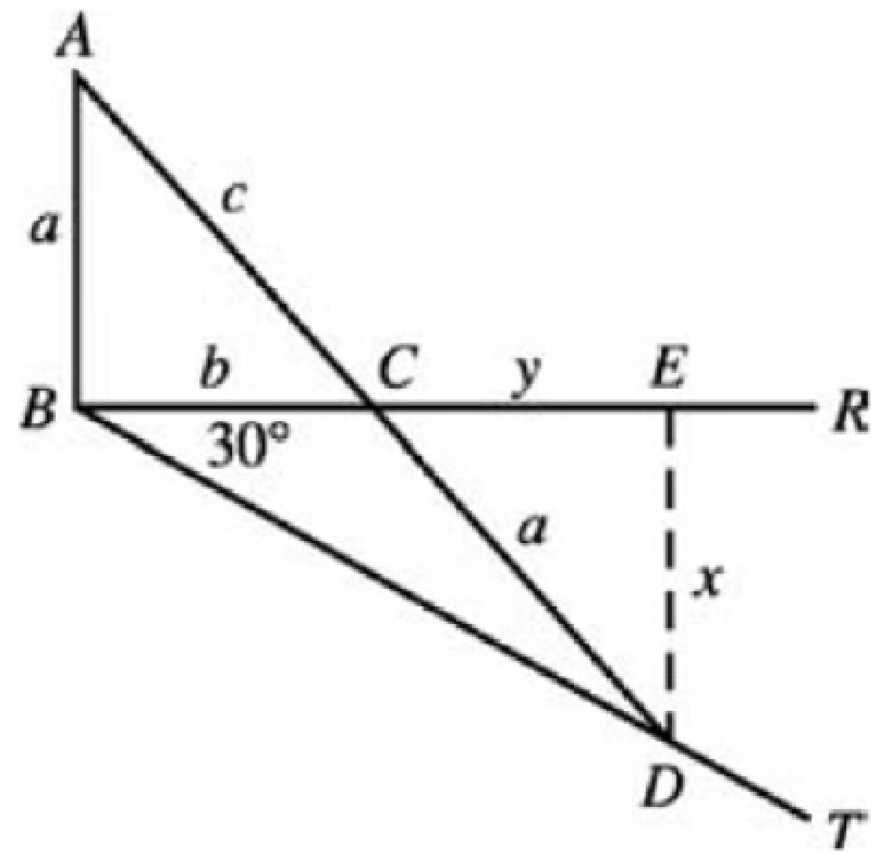
$$\frac{x^2}{a} = \sqrt{2ax}$$

$$\frac{x^4}{a^2} = 2ax$$

$$x^4 = 2a^3 x$$

$$x^3 = 2a^3$$

**7.** Isaac Newton (1642–1727) suggested the following construction for duplicating the cube. Given a segment $AB$, erect a perpendicular $BR$ to $AB$ and draw $BT$ so that angle $ABT$ equals $120°$. Let $D$ be the point on $BT$ such that if $AD$ is drawn meeting $BR$ at $C$, then $CD = AB$.




Establish that if $DE$ is drawn perpendicular to $BR$, each of the following will be true:

(a) Triangles $ABC$ and $DEC$ are similar, whence

$$\frac{a}{x} = \frac{b}{y} = \frac{c}{a}.$$

(b) using $\Delta DEB$, $\text{TAN} = \frac{O}{A} = \frac{DE}{BE}$

$$\frac{1}{\sqrt{3}} = \tan 30° = \frac{DE}{BE}$$

$$= \frac{x}{b+y} = \frac{a^2}{ab+bc}.$$

(c) The result of squaring the last equation and substituting $b^2 = c^2 - a^2$ is

$$c^3(2a+c) = 2a^3(2a+c).$$

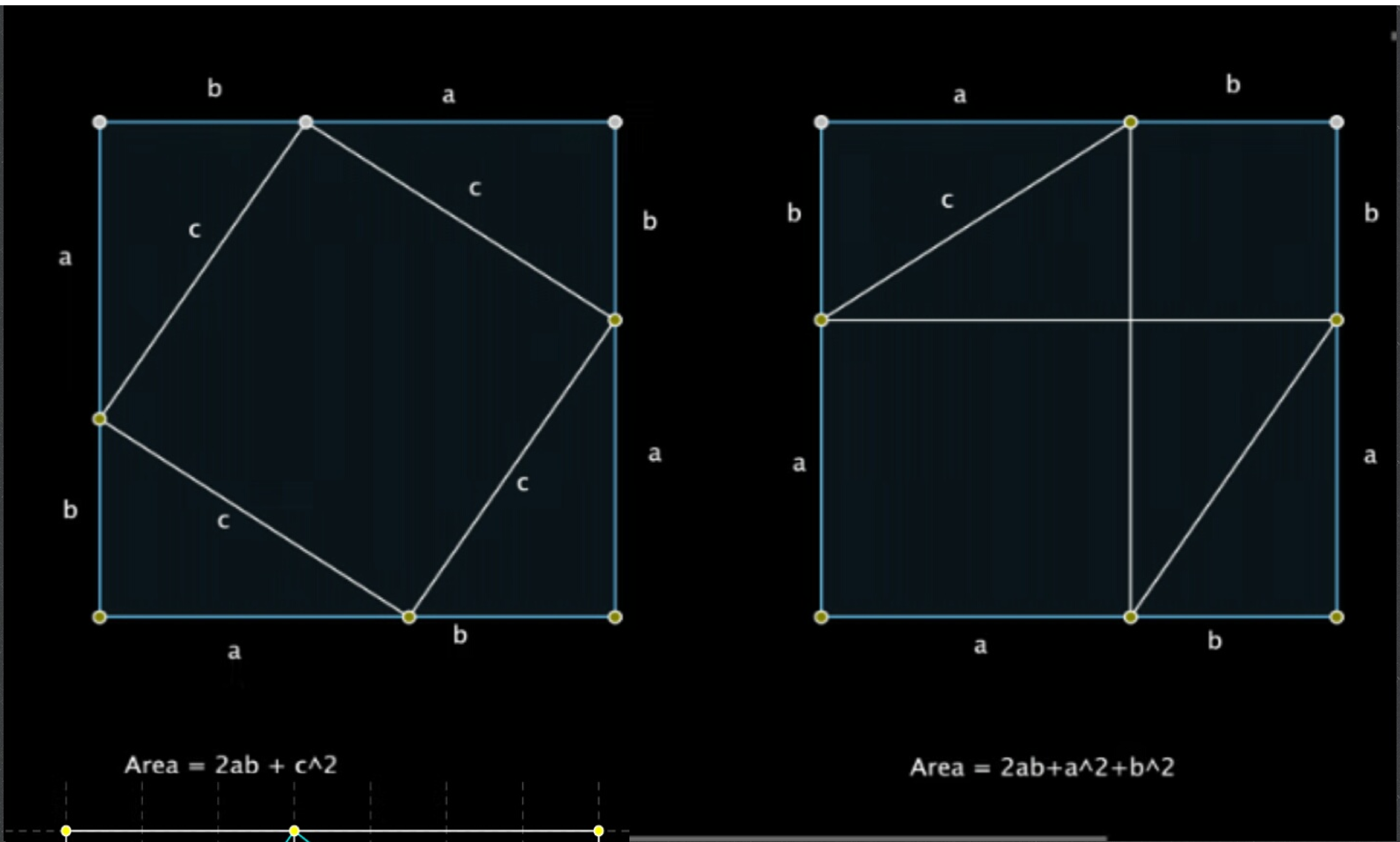
(d) Since $c^3 = 2a^3$, the cube of side $AC$ is double the cube of side $AB$.

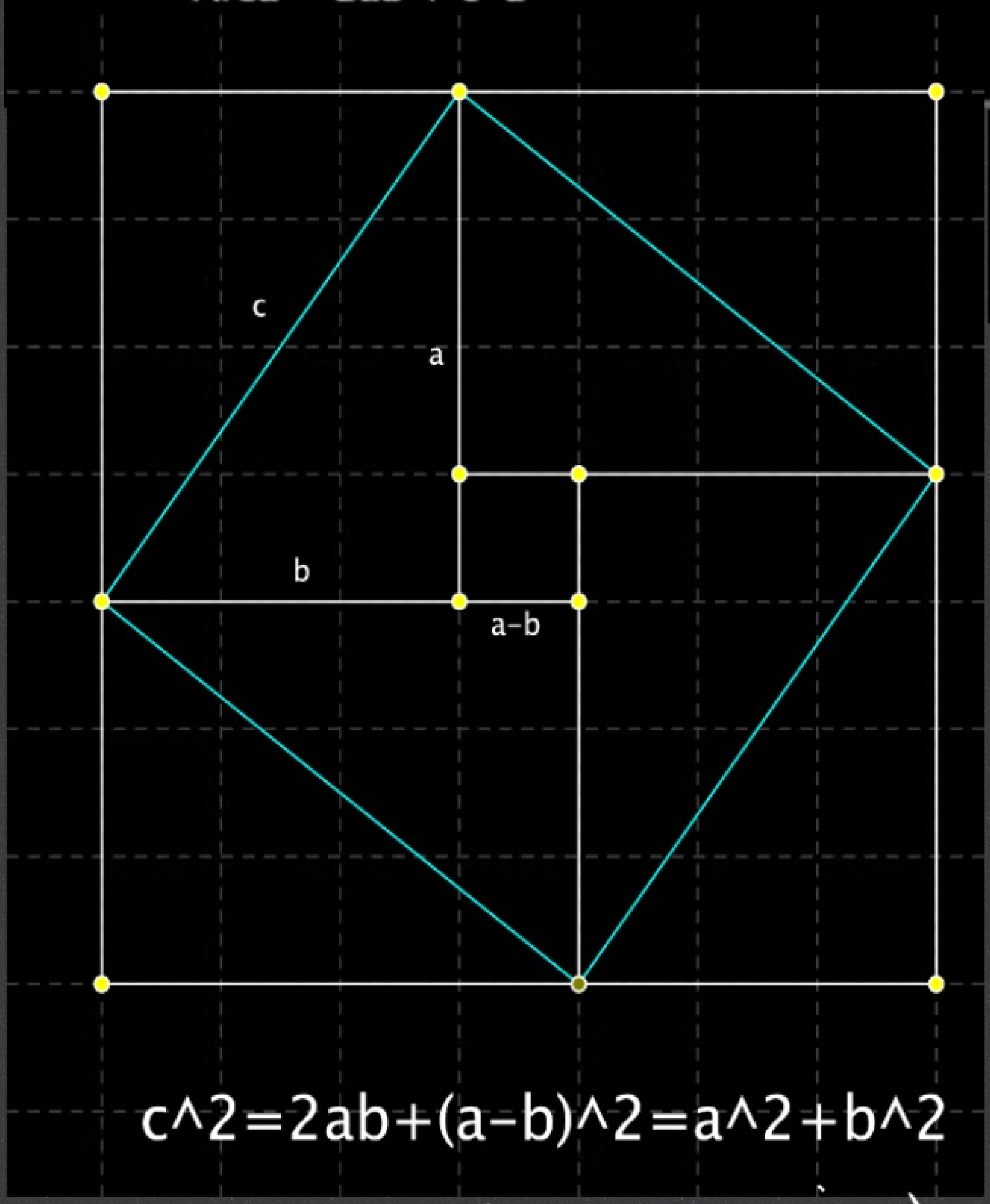
$$\left(\frac{a^2}{ab+bc}\right)^2 = \frac{a^4}{(a^2b^2 + 2ab^2c + b^2c^2)}$$

$$\frac{a^4}{a^2(c^2-a^2) + 2ac(c^2-a^2) + (c^2-a^2)c^2}$$

$$= \frac{a^4}{(c^2-a^2)(a^2+2ac+c^2)}$$

$$= \frac{a^4}{(c^2-a^2)(a+c)^2}$$




$\left(m, \frac{m^2-1}{2}, \frac{m^2+1}{2}\right)$ is a Pythagorean triple

$m \in \{2k \text{ s.t. } k \in \mathbb{Z}\} \to \left(\frac{m^2-1}{2}\right) \in \mathbb{Q}$

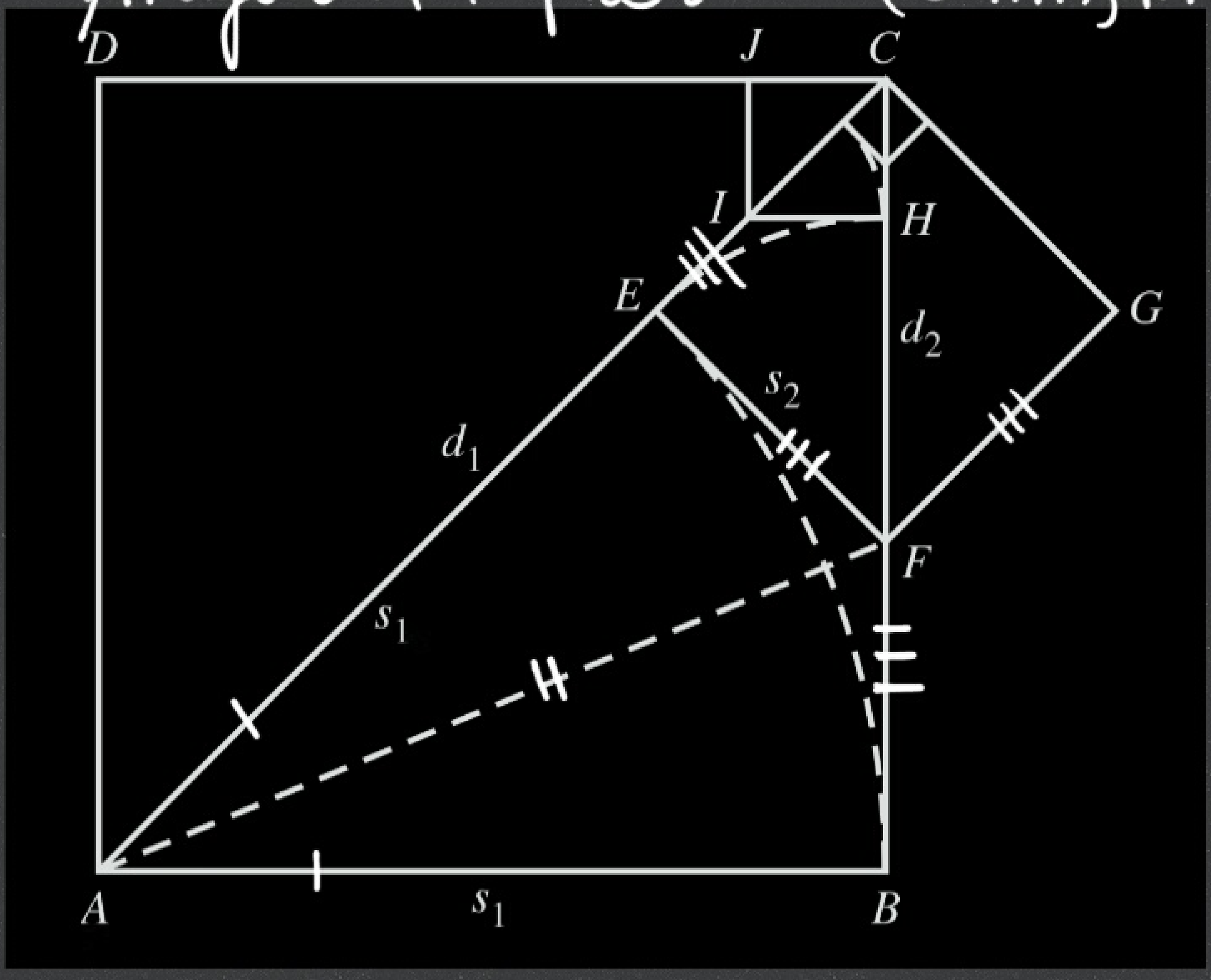
$m \in \{2k+1 \text{ s.t. } k \in \mathbb{Z}\} \to \left(\frac{m^2-1}{2}\right) \in \mathbb{Z}$

Thus $(2n+1, 2n^2+2n, 2n^2+2n+1)$ is a pythagorean triple s.t. $z-y=1\ \forall n$.

$\hookrightarrow (2n+1, 2n^2+2n, 2n^2+2n+1) \subseteq$ PythTriples

Plato was all $(2n, n^2-1, n^2+1)$ is a pythagorean triple

In book X of Elements, Euclid included the complete set of Pythagorean triples: $\vec{\Delta} = (2mn, m^2-n^2, m^2+n^2) \mid m,n \in \mathbb{Z}$ with $m>n$.




$S_2 = d_1 - S_1$

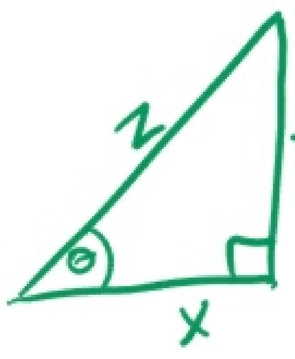
$d_2 = S_1 - S_2$

# History of Mathematics

Skip Lester
April 17, 2014

3.3

**2.** Find all right triangles with sides of integral length whose areas are equal to their perimeters. [*Hint:* The equations $x^2 + y^2 = z^2$ and $x + y + z = \frac{1}{2}xy$ imply that $(x - 4)(y - 4) = 8$.]



$y = z\sin\theta$

$x = z\cos\theta$

$(x-4)(y-4)=8$

$(z\cos\theta - 4)(z\sin\theta - 4) = 8$

Take $x, y, z \in \mathbb{Z}$

$y-4 = 8x-32$

$x = 8y-28 \xleftrightarrow{\text{by symmetry}} y = 8x-28$

$8x - 28 \geq 1$

$8x \geq 29$

$x \geq \frac{29}{8}$ 4ish $\rightarrow x \geq 5$

$x = 8(5) - 28$

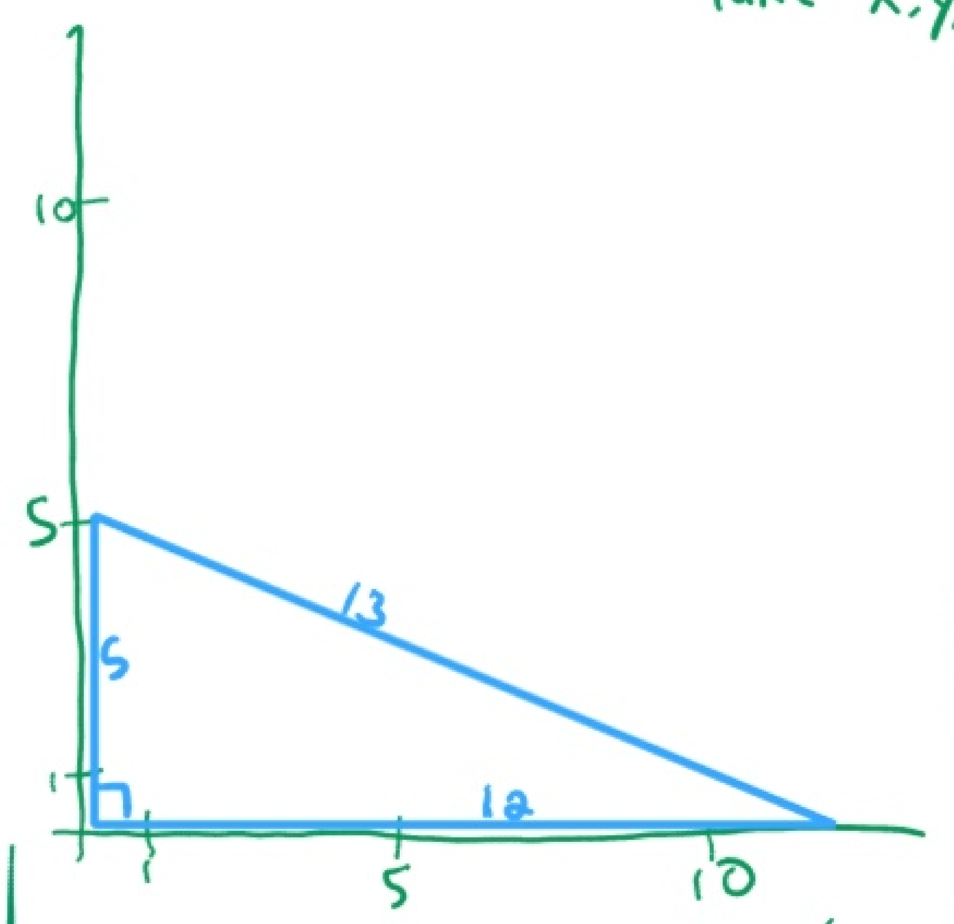
$x = 40 - 28 = 12$

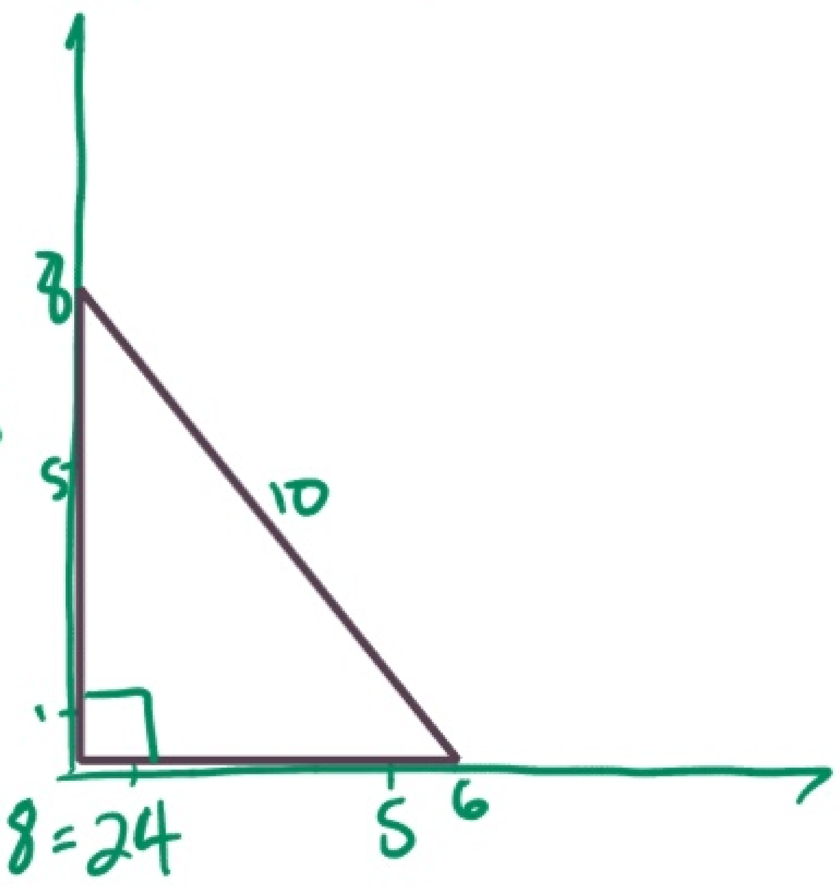
$12^2 + 5^2 = z^2$

$144 + 25 = 169 = 13^2$

$z = 13$
$+ y = 5$
$+ z = 12$
$30$

$\frac{1}{2}(12 \cdot 5) = \frac{60}{2} = 30$





$\frac{1}{2} 6 \cdot 8 = \frac{1}{2} 48 = 24$

$6^2 + 8^2 = 36 + 64 = 100 = 10^2$

$6 + 8 + 10 = 24$ ✓

! $(x-4) \in \mathbb{Z}, (y-4) \in \mathbb{Z} \rightarrow (x-4)(y-4) = 8 \cdot 1$ or $(x-4)(y-4) = 4 \cdot 2$

8 only has two factorizations

$x - 4 = 4$, $x = 8$ $\quad y - 4 = 2$, $y = 6$

We have shown the two corresponding triangles.

---

**4.** Verify that (3, 4, 5) is the only Pythagorean triple involving consecutive positive integers. [*Hint:* Consider the Pythagorean triple $(x, x + 1, x + 2)$ and show that $x = 3$.]

$\{\text{Pythagorean triple}\} = \{x, y, z \in \mathbb{Z} > 0 \wedge x^2 + y^2 = z^2\}$

$(x)^2 + (x+1)^2 = (x+2)^2$

$x^2 + \cancel{x^2} + 2x + 1 = \cancel{x^2} + 4x + 4$

$-x^2 - 4x - 1 \quad -x^2 - 4x - 1$

$x^2 - 2x = 3$

$x(x-2) = 3 \quad x = 3$ or $x - 2 = 3$, $x = 5$

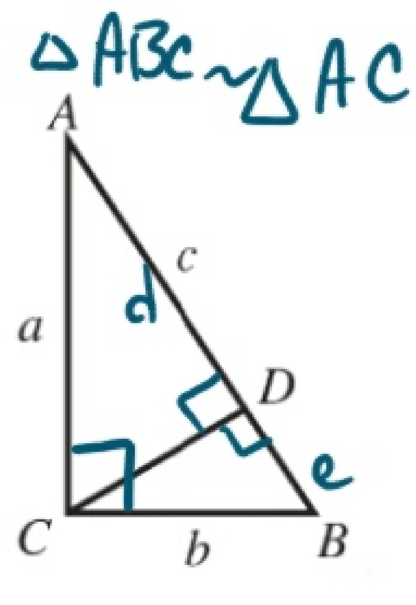
But $5^2 + 6^2 = 25 + 36 = 61 \neq 49 = 7^2$.

Thus $x = 3, y = 4, z = 5$ is the only pythagorean triple of consecutive integers □

---

**15.** A standard proof of the Pythagorean theorem starts with a right triangle $ABC$, with its right angles at $C$, and then draws a perpendicular $CD$ from $C$ to the hypotenuse $AB$.

$\triangle ABC \sim \triangle ACD \sim \triangle CBD$



(a) WTS

$\frac{\overline{AB}}{\overline{AC}} = \frac{\overline{AC}}{\overline{AD}} = \frac{\overline{CB}}{\overline{CD}}$

$\frac{c+e}{a} = \frac{a}{c} = \frac{b}{d}$

$a^2 = c(c+e)$

(a) Prove that triangles $ACD$ and $CBD$ are both similar to triangle $ABC$.

(b) For a triangle $ABC$ with legs of lengths $a$ and $b$ and with hypotenuse of length $c$, use the proportionality of corresponding sides of similar triangles to establish that $a^2 + b^2 = c^2$.

# 4.3 Euclid's Number Theory ~300 B.C. Euclid = Great Organizer

Books VII, VIII, IX of Elements devoted to arithmetic

$a|b \longleftrightarrow \exists c \in \mathbb{Z}$ s.t. $b = ac$

$-13 = (-5)(3) + 2$

$a = -13$ $\quad p = -5 \quad r = 2$

$b = 3$

"a measures b" "b is measured by a"

For $a, b, c \in \mathbb{Z}$, the following are true:

1. $a|0$, $1|a$, $a|a$
2. $a|1$ iff $a = \pm 1$
3. if $a|b$ and $c|d$, then $ac|bd$
4. $(a|b \wedge b|c) \rightarrow a|c$
5. $(a|b \wedge b|a) \leftrightarrow a = \pm b$
6. $(a|b \wedge a|c) \rightarrow a|(bx + cy)$ For any $x, y \in \mathbb{Z}$

Division Algorithm. For $a, b \in \mathbb{Z}$, $b > 0$, $\exists q, r \in \mathbb{Z}$ satisfying $a = qb + r$, $0 \le r < b$

$a = qb + r$

Common Divisors: if $a, b \in \mathbb{Z}$, then $d \in \mathbb{Z}$ is a common divisor of a and b iff $d|a \wedge d|b$.

The largest common divisor of $a \wedge b$ is the G.C.D.

One can use the division algorithm to find the GCD of $a, b \in \mathbb{Z}$.

The last non-zero remainder is the GCD

Euclidean Algorithm

$\gcd(17, 5)$

$17 = (3)(5) + 2$

$5 = (2)(2) + 1$

$2 = (2)(1) + 0$

The last non-zero remainder is 1 so $\gcd(17, 5) = 1$

$\gcd(162, 138) = 6 \rightarrow 162 = (1)(138) + 24$

$138 = (5)(24) + 18$

$24 = (1)(18) + 6$

$18 = 3(6) + 0$

$$\begin{array}{r} 138 \\ \underline{-\;\;24} \end{array} \qquad \begin{array}{r} 5 \\ 24\overline{)138} \\ \underline{120} \\ 18 \end{array}$$

Now $6 = 1 \cdot 24 - 1 \cdot 18$

$1 \cdot 24 - 1[138 - 5 \cdot 24]$

$= 6 \cdot 24 - 1 \cdot 138$

$= 6[162 - 138] - 138$

$6 \cdot 162 - 7 \cdot 138$

$1 = 5 - 2[17 - 3 \cdot 5]$

$1 = -2 \cdot 17 + 7 \cdot 5$

Thus we can write $\gcd(a, b)$ as a linear combination of a and b.

For $a, b \in \{\mathbb{Z} \setminus 0\}$, $\exists x, y$ s.t. $\gcd(a, b) = ax + by$

$17 = 3 \cdot 5 + 2 \quad 2 = 17 - 3 \cdot 5$

$5 = 2 \cdot 2 + 1$

$2 = 2 \cdot 1 + 0 \quad 1 = 5 - 2 \cdot 2$

Definition:

Two integers a, b are said to be Relatively Prime to each other iff $\gcd(a, b) = 1$.

Definition $p \in \{\mathbb{Z} > 1\}$ is prime $\longleftrightarrow (a|p \leftrightarrow a = 1)$

FTA: $\forall n \in \{\mathbb{Z} > 1\}$, n can be expressed as a product of primes

$n = a_0 p_0 + a_1 p_1 + \cdots + a_n p_n$

$\infty$ of Primes

Proof by #

would force p to divide 1.

Initial Contract

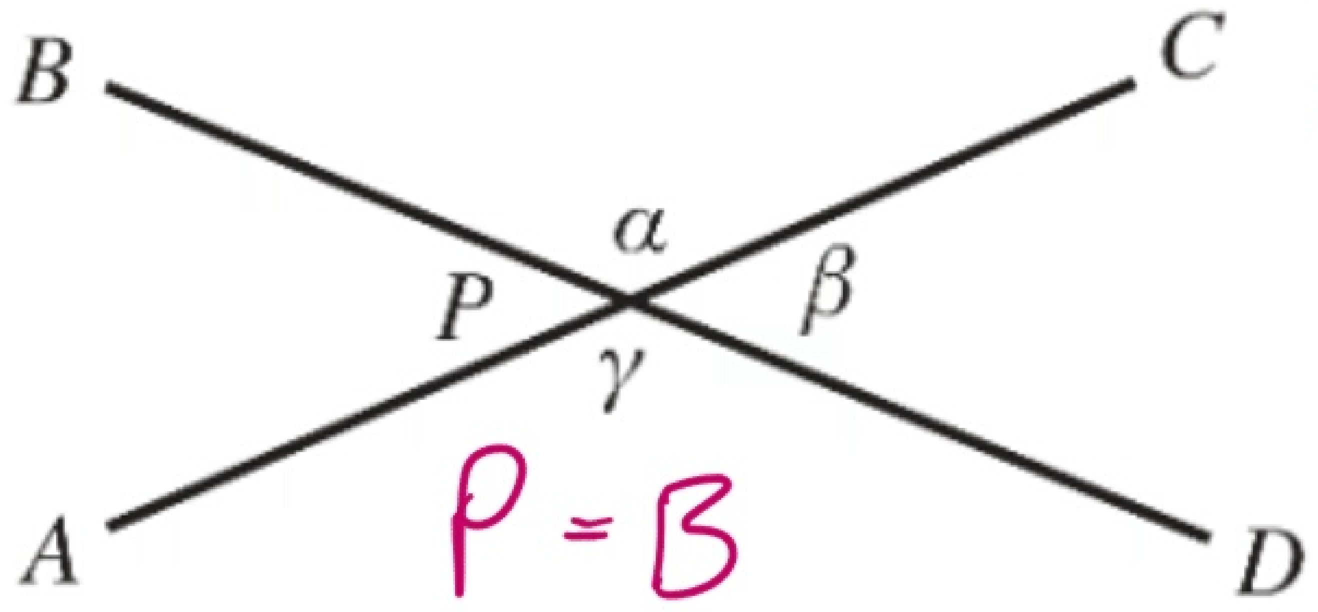
June 3, The paradox of 12.3 in Book set theory

Mike Emerick-Clayton

History → implications
applications
↳ Examples to help explain.

40 minutes

**2.** *Proposition 15.* If two lines cut one another, then they make vertical angles that are equal. [*Hint:* Appeal to Proposition 13, which says that if a ray is drawn from a point on a line, then the sum of the pair of supplementary angles formed is equal to two right angles.]




$P = B$

$180 - b + p = 180$
$-180 \qquad -180$
$-b + P = 0$
$P = b$

$a + P = 180$
$P + y = 180$
$y + B = 180$
$B + a = 180$
$a = 180 - b$

$y + (180 - a) = 180$
$-180 \qquad -180$
$y - a = 0$
$y = a$

Proof: Let $\overline{AC}$ intersect $\overline{BD}$ forming Angles $B, a, P, Y$

Then by proposition 13, $\angle BPA + \angle AYD = 2 \cdot 90° = 180°$ or $\pi$ radians.

But likewise $\angle AYD + \angle DBC = 180°$, and $\angle DBC + \angle CaB = 180°$ and $\angle CaB + \angle APB = 180°$

Then $180 - b + p = 180 \rightarrow B = P \wedge Y = \alpha$ □ QED

Presentation notes: Use CRAP

content
- Presents timeline, topic/mathematicians
- States definitions/Theorems
- Present examples relevant to defs./Theorems
- "Snapshots" of proofs

• Chance to tell interesting story w.r.t. topic.
Quotes

explanations
confidence
- Face to face
- Answers Audience Questions
- Overall communication of main point of topic

Vary voice for dramatic effect

Rehearse & time it (50 minutes)

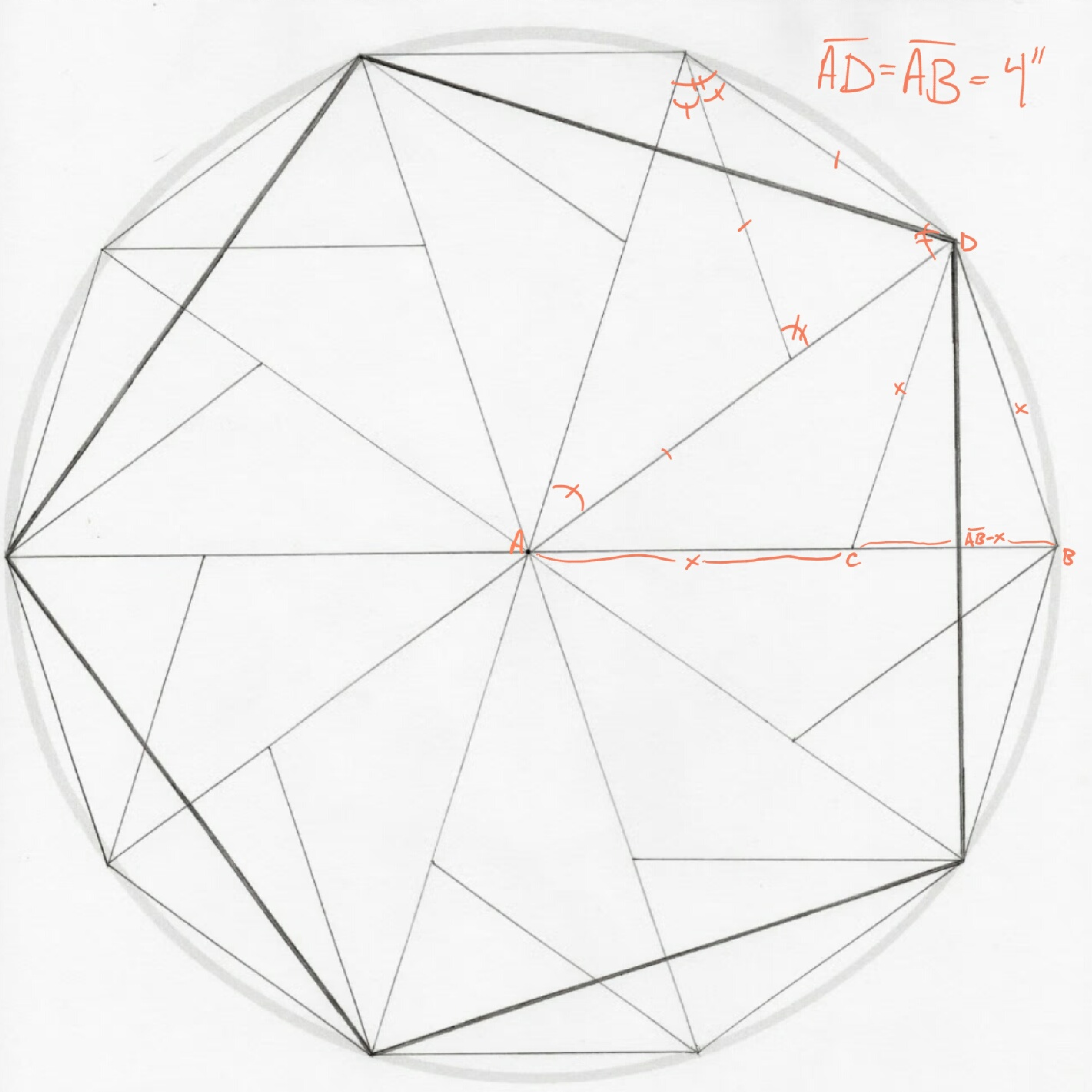
CAn use white board

Conjectures + open problems add interest

Move! Move! Move!

Repeat Big Ideas for comprehension

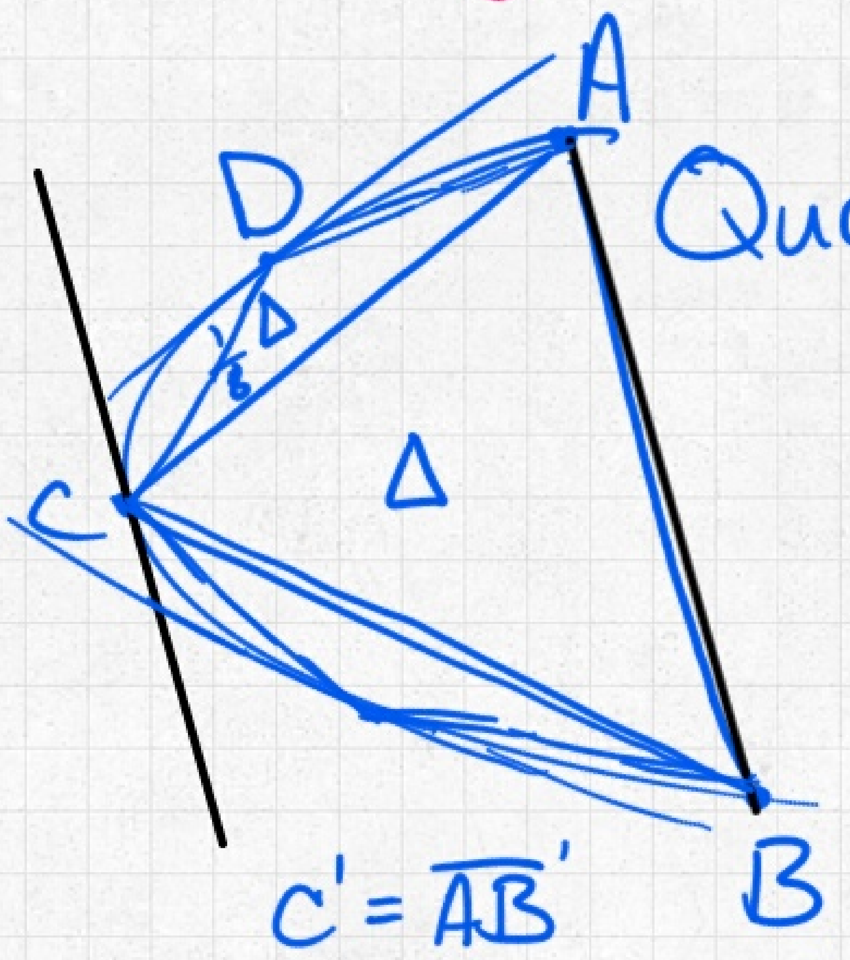
# Construction of an Inscribed Regular Pentagon



4/22/2014 HOM Notes

Circumference $= 2\pi r = \pi(\text{Diameter})$

Estimating Pi → used method of exhaustion → computing perimeter of a circle by computing perimeters of regular polygons with a growing number of sides.




Quadrature of a parabolic segment.

May 8 rough draft of Presentation due

Thursday April 24, 2014 HOM Notes

Leonardo of Pisa (1175-1250) aka Fibonacci aka son of Bonnacio

Rabbit problem in Liber Abaci

Any Sequence can be given by an explicit or recursive formula

Recursive Formula (Albert Girard, 1634)

$F_1 = F_2 = 1 \quad F_n = F_{n-2} + F_{n-1}$ for $n \geq 3$

Explicit Formula (Binet's Formula: Binet (1786-1856))

$$F_n = \frac{\Phi^n - \Psi^n}{\sqrt{5}} \text{ where } \Phi = \frac{1+\sqrt{5}}{2} \quad \Psi = \frac{1-\sqrt{5}}{2}$$

$$\gcd(F_n, F_{n+1}) = 1; \lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \frac{1+\sqrt{5}}{2} = \Phi$$

5.1 & 6.3 discussion

Christians vs Classical knowledge.

**16.** Use the Euclidean algorithm to obtain integers $x$ and $y$ satisfying: $a = qb + r \quad 0 \le r < b \quad a, b \in \mathbb{Z}, b > 0$

(a) $\gcd(56, 72) = 56x + 72y$.
(b) $\gcd(24, 138) = 24x + 138y$.
(c) $\gcd(119, 272) = 119x + 272y$.
(d) $\gcd(1769, 2378) = 1769x + 2378y$.

$272 - 238 = 44 \qquad 119 - 88 = 31$

(a) $\gcd(56,72) \to 56 = 0\cdot 72 + 56$, $\hookrightarrow = 8$

$72 = 1\cdot 56 + 16$
$56 = 3\cdot 16 + 8$
$16 = 2\cdot 8 + 0$

$8 = 56 - 3[72 - 56]$
$8 = -3(72) + 4(56)$
$x = -3 \quad y = 4$

(b) $\gcd(24,138) - 24 = 0\cdot 138 + 24$, $\hookrightarrow = 6$

$138 = 5\cdot 24 + 18$
$24 = 1\cdot 18 + 6$
$18 = 3\cdot 6 + 0$

$6 = 24 - [138 - 5\cdot 24]$
$6 = 6\cdot 24 - 1\cdot 138$
$x = 6 \quad y = 1$

(c) $\gcd(119,272) \to 119 = 0\cdot 272 + 119$

$272 = 2\cdot 119 + 44$
$119 = 2\cdot 44 + 31$
$44 = 1\cdot 31 + 13$
$31 = 2\cdot 13 + 5$
$13 = 2\cdot 5 + 3$
$5 = 1\cdot 3 + 2$
$3 = 1\cdot 2 + 1$
$2 = 2\cdot 1 + 0$

$\gcd(119,272) = 1$
$1 = 3 - [5 - 3] \to 1 = 2[13 - 2\cdot 5] - [31 - 2\cdot 13]$
$2\cdot 3 - 5$

(d) $\gcd(1769,2378) \to 1769 = 0\cdot 2378 + 1769$

$1769 - 1527 = 242 \qquad 2378 - 1769 = 509 \qquad 509 - 484 = 25$

$2378 = 1\cdot 1769 + 509$
$1769 = 3\cdot 509 + 242$
$509 = 2\cdot 242 + 25$
$242 = 9\cdot 25 + 17$
$25 = 1\cdot 17 + 8$
$17 = 2\cdot 8 + 1$
$8 = 8\cdot 1 + 0$

$\gcd(1769, 2378) = 1$

**20.** (a) Find all prime numbers that divide 40! (recall that $40! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots 40$).
(b) Find the prime factorization of the integers 1234; 10,140; and 36,000.

(a) $p \mid 40!$
We know $\forall\, n \in \{0 < n \le 40\} \in \mathbb{Z}$, $n \in$ factor of $40!$
All primes less than 40 therefore divide $40!$
$\hookrightarrow = \{2,3,5,7,11,13,17,19,23,29,31,37\}$ = prime Factors of $40!$

$2535 \div 5 = 507$

(b) $1234 = 2\cdot 617$

$10{,}140 = 2(5{,}070)$
$= 2^2(2535)$
$= 2^2\cdot 5\cdot 507$
$= 2^2\cdot 5\cdot 13^2\cdot 3$

$36000 = 6^2\cdot 10^3$
$= (2\cdot 3)^2 (2\cdot 5)^3$
$= (2^2\cdot 3^2)(2^3\cdot 5^3)$
$2^5\cdot 5^3\cdot 3^2$

$13 \times 9 = 117 \qquad 507 \div 13 = 39 \qquad \frac{39}{13} = 3$